CYLINDRICAL WAVES OF FINITE AMPLITUDE IN A RAREFIED PLASMA

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It is well-known that the profile of finite-amplitude waves in gas-dynamics is determined by non-linear effects and dissipative processes. In a rarefied plasma, where the free path of the particles is substantially greater than the characteristic dimensions, it is often possible to neglect dissipative processes, and the wave profile is formed under the influence of nonlinear and dispersive effects, the latter being connected with a departure from the linear wave dispersion law characteristic of ordinary gas dynamics. Taking dispersion effects into account leads to the "smearing out" of the wave profile, which compensates for twisting as a result of nonlinearity. Thus it is possible for stationary waves of finite amplitude to exist and propagate without changing their form (isolated, periodic and shock waves with oscillatory structure). Stationary waves have been studied fairly fully [1–6].

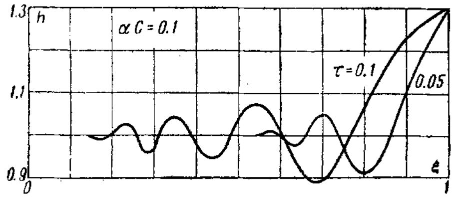

Fig. 1

Nonstationary waves of finite amplitude have been studied in a series of papers. For example, in [7] one of the authors investigated plane nonstationary waves of finite, but small amplitude, propagating in a cold plasma both transverse to, as well as at an angle to a constant magnetic field (such waves have a different dispersion law). In [8, 9] plane waves of finite amplitude were investigated by means of numerical integration, neglecting dissipative processes.

The present paper carries out the calculation of nonstationary cylindrical waves propagating in a rarefied cold plasma situated in a strong magnetic field.

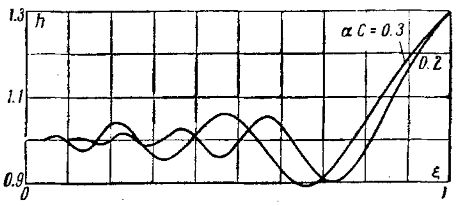

Fig. 2

The calculations were made on the assumptions formulated in [7], namely, the plasma is quasi-neutral $N_i = N_e$, the kinetic gas pressure is neglected in comparison with the magnetic pressure $p \ll H^2/8\pi$, and collisions between particles are also neglected. When t = 0, the plasma with density N = const fills a cylinder of radius a and is situated in a uniform magnetic field H_0, directed along the axis of the cylinder. Subsequently, the magnetic field on the boundary of the plasma cylinder begins to increase rapidly according to some law. As a result of this, magnetic perturbations propagate towards the axis of the cylinder and the plasma column begins to be compressed under the action of the increasing magnetic pressure. Under the conditions of the experiments [10] the propagation of cylindrical waves is characterized by positive dispersion (see [7]), i.e., the harmonics with shorter wavelength precede the main front, which moves with roughly the Alfvèn velocity, in other words, the leading wave front has an oscillatory structure. This is associated with the fact that under the conditions of the experiments [10] the "magnetic piston" moves in a direction which is not strictly perpendicular to the constant magnetic field. We thus introduce into the equations a certain effective angle α between the plane of the piston and the magnetic field H_0. Since in the process of moving the mass of plasma remains constant, and also the magnetic pressure on the plasma-vacuum boundary is given, the calculation is most conveniently carried out in Lagrangian coordinates. In addition, we shall assume that all quantities depend only on the radius r and the time t (one-dimensional problem). We may then write the basic system of equations in dimensionless variables in the form

$$\frac{\partial u}{\partial \tau} - \frac{v^2}{x} = -\frac{1}{2}\frac{x}{\xi}\frac{\partial}{\partial \xi}(h_1^2 + h_2^2) - \frac{V}{x}h_2^2, \qquad V = \frac{x}{\xi}\frac{\partial x}{\partial \xi},$$

$$\frac{\partial v}{\partial \tau} + \frac{uv}{x} = \alpha\left(\frac{V}{x}h_2 + \frac{x}{\xi}\frac{\partial h_2}{\partial \xi}\right), \qquad \frac{\partial w}{\partial \tau} = \alpha\frac{x}{\xi}\frac{\partial h_1}{\partial \xi}, \qquad \frac{\partial x}{\partial \tau} = u,$$

$$\frac{\partial}{\partial \tau}(Vh_2) - \alpha\frac{x}{\xi}\frac{\partial v}{\partial \xi} - \frac{V}{x}h_2 u = \alpha C\frac{x}{\xi}\frac{\partial}{\partial \xi}\left(\frac{x}{\xi}\frac{\partial h_1}{\partial \xi}\right), \qquad (1)$$

$$\frac{\partial}{\partial \tau}(Vh_1) - \frac{\alpha}{\xi}\frac{\partial}{\partial \xi}(xw) = -\frac{\alpha C}{\xi}\frac{\partial}{\partial \xi}\left(Vh_2 + \frac{x^2}{\xi}\frac{\partial h_2}{\partial \xi}\right),$$

$$V = \frac{N_0}{N}, \qquad C = \frac{c}{\omega_{0i}}, \qquad \omega_{0i} = \frac{\sqrt{4\pi N_0 e^2}}{\sqrt{M}}, \qquad V_a = \frac{H_0}{\sqrt{4\pi N_0 M}}.$$

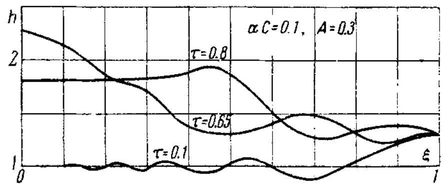

Fig. 3

Here u, v, w are the radial, azimuthal and longitudinal velocity components, h_1, H_2 the longitudinal and azimuthal magnetic field components, V the specific volume (N_0) is the initial density), V_0 the Eulerian coordinate, and ξ, the Lagrangian coordinate. Velocities are normalized by the Alfvèn velocity V_a, corresponding to the initial values of density and magnetic field, lengths are normalized by the radius of the cylinder a, the magnetic field by the initial field H_0. The initial conditions are

$$x(\xi, 0) = \xi, \qquad u(\xi, 0) = v(\xi, 0) = w(\xi, 0) = h_2(\xi, 0) = 0,$$

$$h_1(\xi, 0) = V(\xi, 0) = 1. \qquad (2)$$

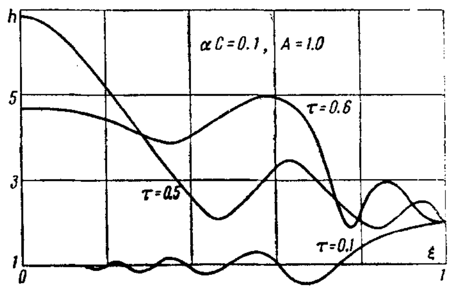

Fig. 4

The boundary conditions are

$$u(0, \tau) = v(0, \tau) = 0, \; h_2(0, \tau) = 0, \; \frac{\partial h_1}{\partial \xi}(0, \tau) = 0, h_2(1, \tau) = 0,$$

$$h_1(1,\tau) = 1 + A f(\tau) \quad (A = \text{const}). \qquad (3)$$

Here $f(\tau)$ is a given function of time.

The system of equations (1), together with conditions (2) and (3), was solved numerically in finite differences, while in the solution the boundary function was taken in the form $f(\tau) = 1 - \exp(-\omega\tau)$. Questions connected with the method of calculation will be dealt with separately. We only note here that the calculation was explicit, the time step being chosen from a knowledge of the stability of the difference method, while at large amplitudes continuous transition through the discontinuities was achieved by introducing the artificial viscosity proposed by Neyman and Richtmeyer [11].

Figures 1–4 give some of the results obtained. We were basically interested in the structure of the magnetic field when it met and was reflected from the axis of the cylinder. Figure 1 shows the propagation of the magnetic disturbance for times which are not large. In accordance with the positive law of dispersion the leading front of the wave has an oscillatory character, while the total field in some places is less than the unperturbed field "rarefaction" wave. The dimension of these oscillations is equal in order of magnitude to the dispersion length $c\alpha/\omega_{0i}$, and their amplitude grows in time.

Figure 2 shows the wave profile shortly after the field begins to increase on the boundary of the plasma filament for two different dispersion lengths. The longer the dispersion length, the higher the oscillations, their amplitude is somewhat larger and their number is also greater. Figures 3 and 4 give the profile of the magnetic field at the time most suitable for revealing the characteristic stages of the process for two values of the amplitude A of the magnetic field on the plasma-vacuum boundary. The main wave front moves with roughly the Alfvèn velocity V_a, which naturally increases as the amplitude of the magnetic field on the boundary increases.

It is clear from Figs. 3 and 4 that for comparatively short times when the main front has not yet reached the axis, the leading oscillations have dimensions of the order of the dispersion length, and the amplitude is fairly large.

At subsequent moments of time the wave velocity increases. Insofar as the field strength increases as the axis of the cylinder is approached, a cumulative process occurs in which the magnetic field close to the axis increases significantly compared with the field on the boundary of the plasma filament, and subsequently reflection of the wave from the axis takes place. The motion of the reflected wave accompanied by the leading oscillations was traced almost to the moment of "collision" with the moving plasma boundary.

Calculations are now being carried out for conditions which are as close as possible to those which exist in experiments.

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